WAVE EFFECTS IN A TWO-PHASE MEDIUM

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A wave equation derived in the article is used to obtain a relation for the propagation velocity of small perturbations in a two-phase medium. This relation is confirmed indirectly by a computer analysis of a wet-steam nozzle, as well as experimentally.

In the flow of a two-phase medium, as in the case of a continuous medium, critical effects take place in connection with the characteristics of the propagation of small perturbations.

Many papers have been published on the propagation velocity of small perturbations in a two-phase medium (see the bibliography in [1]). In the majority of cases, however, the initial assumption is invoked that the following equation is valid in the two-phase medium:

$$C^2 = \frac{dP}{d\rho}. \tag{1}$$

In this case the two-phase medium is replaced by a nominal continuum of density ρ without any velocity differential between the gaseous and liquid phases. In practice, as for example in the case of wet steam flow in nozzles, the settling of liquid drops from the vapor is observed. Consequently, two distinct velocities result: the velocity of the vapor phase and the velocity of the drops. Moreover, inasmuch as relation (1) is deduced from an analysis of the wave equation derived for a continuum, there is no obvious justification for its application to the two-phase case. It is sensible, therefore, to consider the propagation velocity of small perturbations by analysis of a wave equation derived for a two-phase medium on the basis of the following assumptions, which are also fundamental to all ensuing arguments in the present article.

1. The real two-phase medium, which consists of a dry saturated vapor and liquid drops, is replaceable by a continuum consisting of a liquid phase having a nominal density $\rho_L = m_L/V$ and a gaseous phase having a nominal density $\rho_V = m_V/V$. This approach is not only admissible in the case of drops of equal diameter distributed uniformly in space, but also in the presence of heterogeneous drops spaced at unequal distances. In the latter case the interaction of the drops and vapor can be treated by the introduction of a certain average drop diameter.

2. The flow of the two-phase medium is assumed to be steady. Therefore, at every point of space any variation of the drop velocity relative to the vapor is elicited solely by the propagation of small perturbations.

3. The variation of the moisture content due to small perturbations is negligible. This assumption is fully justified, because the small perturbation propagation velocity depends on the inertial properties of the medium, i.e., on the total mass of the vapor and drops in the analyzed volume.

4. Due to the absence of adequate experimental data on the boundary layer associated with the flow of a two-phase medium the motion is described by a one-dimensional model, and the influence of the walls is therefore neglected.

5. It is assumed that the vapor phase obeys the Clapeyron–Clausius equation. The latter is satisfied for a high moisture content, i.e., in the presence of a well-developed over-all liquid phase surface, which is conducive to nonrelaxing phase transitions. This assumption is supported in practice by the coincidence of the flow pressure and temperature with the corresponding parameters of the saturated vapor.

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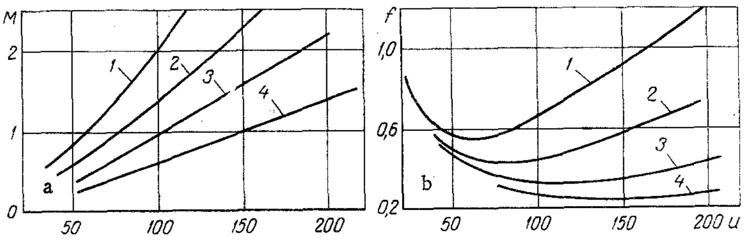

Fig. 1. Variation of the number M (a) and the relative cross section f along a nozzle (b) versus the effective moisture content u", m/sec, for $d_d = 1 \cdot 10^{-4}$m. 1) y = 0.95; 2) 0.9; 3) 0.8; 4) 0.6.

6. The liquid phase is assumed to be incompressible.

For the derivation of our wave equation [2] we consider a small perturbation stimulated in the two-phase medium by the displacement of vapor particles over a distance l(X, t) and, as a result, the displacement of the drops by an amount S(X, t) (in relative motion). Then a layer of the medium of thickness ΔX occupies a new position with coordinates $X + l(X, t)$ and $X + \Delta X + l(X + \Delta X, t)$. The pressure in this case changes by an amount $P_c = P_v - P_0$, and the density of the medium by an amount $\rho_{vc} = \rho_v - \rho_0$, where the subscript 0 refers to the initial state. Due to the smallness of P_c and ρ_{vc} it may be assumed that $P_c = K\rho_{vc}$, where $K = (dP/d\rho_v)_0$.

Since the mass of the vapor in the layer of thickness ΔX does not change, we can write the following with respect to unit area perpendicular to the X axis:

$$\rho_{v0}\Delta X = \rho_v\left[X + \Delta X + l(X - \Delta X,\ t) - X - l(X,\ t)\right]. \tag{2}$$

Inasmuch as ΔX, is small, neglecting second-order small quantities and instituting suitable transformations, we obtain

$$\rho_{vc} = -\rho_{v0}\frac{\partial l}{\partial X}. \tag{3}$$

Next we consider the forces acting on the vapor layer of thickness ΔX. Due to the smallness of the latter resultant pressure is

$$R_f = P(X,\ t) - P(X + \Delta X,\ t) = -\frac{\partial P}{\partial X}\Delta X.$$

Recognizing that X depends only on P_c, we obtain

$$R_f = -\frac{\partial P_c}{\partial X}\Delta X. \tag{4}$$

It is clear that the resultant pressure must be equalized by the inertial forces of the mass of the vapor phase and the mass of the liquid contained in the layer ΔX. Therefore, taking relation (3) into account, we finally have

$$\frac{\partial^2 l}{\partial t^2} = K\frac{\partial^2 l}{\partial X^2} - \frac{\rho_{L0}}{\rho_{v0}}\frac{\partial^2 S}{\partial t^2}. \tag{5}$$

This equation is an inhomogeneous wave equation valid for a two-phase medium. As we know, it does not admit a solution in the form of an arbitrary traveling waveform, so that, expanding the small perturbation in a Fourier series, we can speak of wave dispersion. In the given case the small perturbation velocity depends on the ratio $\varphi(X, t) = S(X,t)/l(X,t)$. Examining the process at a definite time, we write Eq. (5) in a form that coincides with that of a homogeneous wave equation:

$$\frac{\partial^2 l}{\partial t^2} = \left(\frac{K\rho_v}{\rho_v + \rho_L\varphi}\right)\frac{\partial^2 l}{\partial X^2}. \tag{6}$$

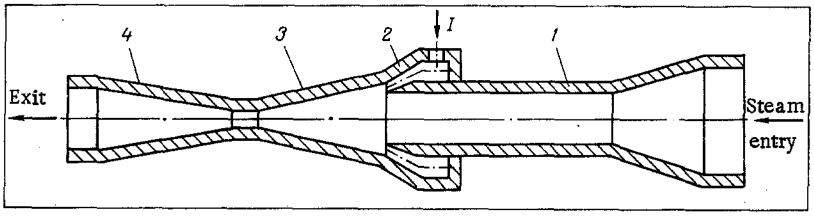

Fig. 2. Schematic of injector. 1) Steam nozzle; 2) liquid nozzle; 3) mixing chamber; 4) divergent channel; I) cooling-liquid entry.

The small perturbation propagation velocity is therefore

$$C=\sqrt{\frac{K\rho_v}{\rho_v+\rho_L\varphi}}=\sqrt{\frac{\frac{dP}{d\rho''}}{1+\varphi y_m^*}}. \tag{7}$$

Inasmuch as

$$\rho_v=\frac{\rho'\rho''}{\rho'+\rho'' y_m^*},$$

we ultimately obtain

$$C^2=\frac{\left(1+\frac{\rho''}{\rho'}y_m^*\right)^2\frac{dP}{d\rho''}}{1+\varphi y_m^*}. \tag{8}$$

The value of the derivative $dP/d\rho''$ is known for most gases and vapors under the condition of an adiabatic process in the perturbation wave.

Applying the Clapeyron–Clausius equation $dP/dT \approx r\rho''/T$ and the equation of state $P=\rho'' RT$ to the vapor phase, we have

$$\frac{dP}{d\rho''}=\frac{r}{\frac{r}{RT}-1}. \tag{9}$$

It is important to note that the values calculated for the derivative $dP/d\rho''$ according to Eq. (9) and for an adiabatic process [3] do not differ by more than 2 or 3%.

Despite the fact that Eq. (8) was obtained for a two-phase medium having a drop structure, i.e., under the condition that β (volume ratio of the vapor to the liquid phase) is clearly greater than unity, it is nevertheless instructive to analyze the variation of the quantity C over the entire range of variation of the moisture content.

Applied to steam, this analysis reveals that for a temperature of 50°C, $\varphi=1$, and $\beta\approx 1$ the small perturbation velocity has a minimum equal to 7.5 m/sec. In the dry vapor region $C^2=dP/d\rho''$, and in the liquid region $C=\infty$; this result is a consequence of the presumed incompressibility of the liquid phase.

Consequently, the transfer of momentum between the drops and gaseous medium in which the perturbations are actually propagating tends to reduce their velocity due to the increase in the inertial properties of the medium. Therefore, in the exit flow of wet steam from a Laval nozzle the velocity of the vapor phase must be equal to the small perturbation velocity in the critical (minimum) cross section of the nozzle, according to Eq. (8).

For a sufficiently large pressure differential in the divergent part of the nozzle the vapor phase velocity can in a certain cross section exceed the velocity of sound, which is equal to $\sqrt{(dP/d\rho'')}$.

The initial assumptions enable us to determine analytically the parameters of steam flowing along a Laval nozzle. We make use of the conservation equations derived for two-phase media in [4].

If we assume that there are no phase transitions, i.e., that $y=\text{const}$, then the following equations are required in addition to the Clapeyron–Clausius and state equations:

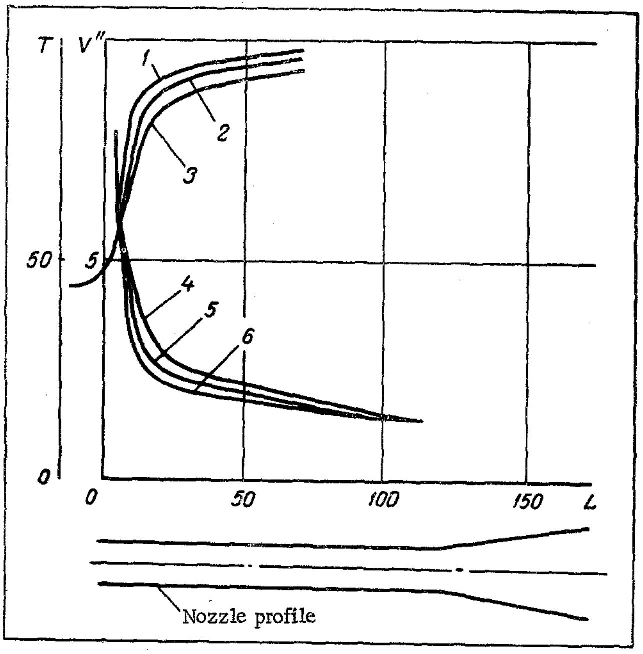

Fig. 3. Variation of the temperature (curves 1-3) and the corresponding specific volume of the vapor phase (4-6) along a nozzle versus the volumetric flow of vapor (T in °C, V in m^3/kg; L in mm). 1, 6) G'' = 0.0075 kg/sec; 2, 5) 0.012; 3, 4) 0.02 kg/sec.

1) the equation of state in integral form

$$G = \frac{F\rho'' u'' (1 + y^*)}{1 + \zeta}, \tag{10}$$

in which $\zeta = y^*\rho''/\theta\rho'$ denotes the obstruction factor of the drop channel; since the liquid density is much greater than the vapor phase density, we assume hereinafter that $\zeta = 0$;

2) the equation of motion

$$\begin{aligned} 2(1 - y_m)\rho u'' du'' + 2y_m \rho u' du' + [(u'')^2 (1 - y_m) \\ + (u')^2 y_m]\, d\rho - [(u'')^2 - (u')^2]\, \rho\, dy_m = -dP; \end{aligned} \tag{11}$$

3) the phase interaction equation

$$m_d \frac{du'}{dt} = C_d S_d \frac{\rho'' (\bar{u})^2}{2} \tag{12}$$

in which

$$C_d = C_i \mathrm{Re}^{-0.5}; \quad \bar{u} = u'' - u'.$$

According to published data [4, 5], the coefficient C_i can be assumed to be equal to 12.5.

The assumption of a constant effective moisture content along the nozzle is admittedly rather coarse. However, our sole purpose in the given analysis is to compare the velocity of the phases in the critical section with the small perturbation propagation velocity calculated according to (8), which involves the mass rather than the effective moisture content, so the stated assumption does not affect the final results.

Equations (10), (11), and (12) were solved numerically on a digital computer by the Runge–Kutta method for various initial values of y and a constant drop diameter. The results, which are given in Fig. 1, indicate that the number M, defined as the ratio of the vapor phase velocity to the small perturbation velocity calculated according to (8), is equal to unity in the critical section. The coefficient φ is then assumed to be equal to unity, and the value of $dP/d\rho''$ is calculated according to (9). An analogous result is obtained for any selected drop diameter.

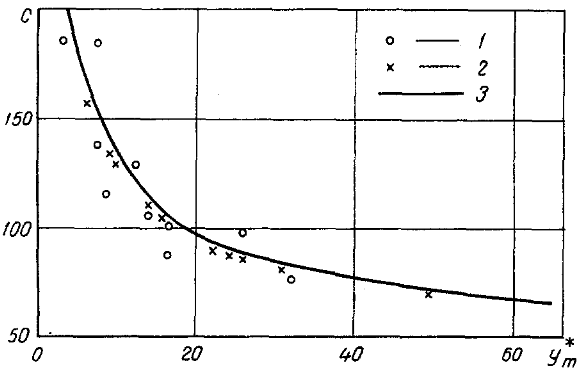

Fig. 4. Small perturbation propagation velocity in a two-phase medium (m/sec). 1) Experimental points; 2) calculated (for a nozzle); 3) theoretical dependence [Eq. (8)].

The foregoing theoretical conclusions were corroborated experimentally in a test of a steam injector, illustrated schematically in Fig. 2. It consists of a convergent nozzle having a long (120 mm) cylindrical part, plus a mixing chamber into which the cold liquid from the divergent channel is admitted. The ratio of the length of the cylindrical part of the nozzle to its inside diameter is twelve. We measured the vapor and cold liquid flows, the pressure and temperature at the entry and exit of the injector, and, using a movable thermocouple set up on the axis of the through-flow section, the temperature at any point of the nozzle. Also, we measured the flow pressure along the nozzle at three check points. Under any conditions the pressure at these points corresponded within 1 or 2% to the saturation temperature measured with the movable thermocouple.

Characteristic curves of the temperature on the nozzle axis according to the movable thermocouple data and the corresponding curves of the specific volumes of saturated steam are shown in Fig. 3.

The abrupt drop in temperature to the temperature in the mixing chamber over a length of 20 to 30 mm in the end part of the nozzle is typical of all sets of conditions. A calculation of the vapor phase velocity in the nozzle exit section according to these data yields a supersonic regime with respect to the vapor phase.

Therefore, taking into account the cylindrical shape of the nozzle, in the calculations we assume that the vapor phase velocity at the nozzle exit is equal to the local velocity of sound. Plotting the curve of the specific volume variation of the vapor along the nozzle makes it possible to determine the site at which this quantity begins to change abruptly.

As apparent from Fig. 4, the vapor phase velocity at that site turns out to be equal to the small perturbation velocity according to Eq. (8).

Returning to the analysis of the curves of Fig. 3, we must point out that the behavior of the vapor phase at the end of the nozzle attests to the fact that the rarefaction wave front propagating upstream from the mixing chamber is indistinct and is situated at a length of about 20 to 30 mm.

The analytical description of the given effect, which is associated with the drop structure of the flow, is based on the fact that in the course of propagation the momentum of the perturbation wave decreases uniformly due to mechanical interaction with the drops. The mechanical interaction length can be roughly determined by analogy with the propagation of light in a dust atmosphere.

We consider a two-phase layer of unit area between the coordinates Z and Z + dZ. This layer contains NdZ drops. Let the momentum q acquired by a drop be directly proportional to its middle cross section ($q = \alpha S_d$). Then in the layer dZ momentum transfer takes place, equal to

$$dq = qN\alpha S_d dZ.$$

From this relation, integrating, we obtain

$$\frac{q}{q_0} = \exp(-N\alpha S_d Z). \tag{13}$$

Relation (13) determines the behavior of the flow parameters in the end section of the nozzle. The coefficient α can be determined experimentally from the curves of Fig. 3.

For approximative calculations we can find the mechanical interaction length on the basis of straightforward arguments.

Let the entire momentum of the perturbation wave be acquired by the drops over a length L. In the two-phase medium we separate out a cylinder of length L and base area F. Then the volume LF contains NLF drops. Let the total area of the middle cross sections of all the drops be equal to F at the length L, i.e., let

$$S_d NLF = F,$$

whence

$$L = \frac{1}{S_d N}. \tag{14}$$

Inserting the value of N, equal to

$$N = \frac{m_d}{\frac{1}{6}\pi d_d^3 \rho'} = \frac{\rho'' \frac{y_m}{1 - y_m}}{\frac{1}{6}\pi d_d^3 \rho'},$$

into expression (14) and transforming appropriately, we obtain

$$L = \frac{2}{3}\rho' u' \frac{F_n}{G''} \cdot \frac{d_d}{y}. \tag{15}$$

If we assume that the diameter and velocity of the drops remain invariant over the length L, we can use the experimental data to calculate the mechanical interaction length. For example, in the case of curve 3 of Fig. 3 we have L = 12 mm for d_d = 0.06 mm. This estimate implies order-of-magnitude agreement with experiment.

NOTATION

C	is the small perturbation propagation velocity;
P	is the pressure;
ρ	is the total density of the medium;
t	is the time;
X	is the coordinate;
V	is the analytical geometric volume;
m_L	is the liquid mass;
m_V	is the vapor mass;
y^*_m	is the liquid to vapor mass ratio in the analytical volume;
ρ'	is the liquid density;
ρ''	is the vapor density;
T	is the absolute temperature;
R	is the gas constant;
r	is the phase transition heat;
m_d	is the drop mass;
G	is the mass flow of wet vapor;
F	is the channel cross section;
u''	is the vapor phase velocity;
u'	is the liquid phase velocity;
$\theta = u'/u''$	is the slip factor;
y^*	is the effective moisture content relative to the volumetric vapor flow;
y_m	is the mass flow of moisture;
Re	is the Reynolds number for flow past a drop;
d_d	is the drop diameter;
ν''	is the vapor kinematic viscosity;
S_d	is the middle cross section of a drop;
Z	is the coordinate coinciding with the nozzle axis;
q	is the perturbation wave momentum per unit cross section;
F_n	is the nozzle exit cross section;
G''	is the volumetric vapor flow;
f	is the relative nozzle cross section.

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